PHYS 2LA: Lab 3
Projectile Motion
(Includes Pre-Lab Assignment)

Objectives

In the first two labs, you studied objects in free-fall, i.e. objects moving in one dimension parallel to the force of gravity. Today, you will complicate that system by adding a second dimension of motion. In today’s experiment, you will study 2D projectile motion. This lab is specifically designed to develop your skills in applying physical models to predict future observations, in analyzing the relation between plotted data and modelled behavior, and in scientific communication.

NOTE: Vectors will be used increasingly often as you move forward in the physics sequence – basically any time you study systems in more than 1D. If you need to review how to notate, add, subtract, and manipulate vectors, please do so now!!

For the following activities, you will use a physics simulation program. Visit: https://uglabs.physics.ucr.edu/ for lab downloads and links.

Motivation

What makes projectile motion such an interesting phenomenon to study at the beginning of an introductory physics course? It’s not necessarily that it’s so profound that we think students will drop everything and love physics after learning how to analyze this motion. What makes projectile motion interesting is that it is a commonly observed phenomenon that can be modeled extremely precisely with minimal complicated math required.

In other words, based on observations of projectile motion in real life, we have developed a mathematical and physical model that allows us to predict the trajectory of any projectile motion as long as we know some initial parameters about the system, like launching speed or launching angle. The physical model is
also powerful enough to allow us to back-calculate these initial parameters based on measurable properties associated with the flight, like time of flight, horizontal range of flight, or even maximum vertical position of the projectile in-flight.

**Introduction**

For a projectile launched with an initial speed, \( v_0 \), and angle, \( \theta \), above horizontal, the x- and y-components of the initial velocity are:

\[ v_{0x} = v_0 \cos \theta \quad \text{(Eq. 1)} \]
\[ v_{0y} = v_0 \sin \theta \quad \text{(Eq. 2)} \]

Due to the constant downward acceleration of gravity, \( g \), the kinematic equations of motion are:

\[ \text{Range} = x - x_0 = v_{0x} t \quad \text{(Eq. 3)} \]
\[ y - y_0 = v_{0y} t - \frac{1}{2} gt^2 \quad \text{(Eq. 4)} \]

where \( x_0 \) is the starting horizontal position, \( y_0 \) is the starting height, \( x \) is the final horizontal position, \( y \) is the final height, and \( t \) is the total time of flight.

If we take the assumption that the final \( y \)-position is equal to the initial \( y \)-position \( (y = y_0) \), then we can rearrange these equations to solve for time-of-flight and range in terms of the projectile’s initial velocity and launching angle:

\[ t = \frac{2v_0}{g} \sin \theta \quad \text{(Eq. 5)} \]
\[ \text{Range} = x - x_0 = \frac{2v_0^2}{g} \sin \theta \cos \theta = \frac{v_0^2}{g} \sin 2\theta \quad \text{(Eq. 6)} \]

**Simulation Apparatus:**

This simulation will allow you to blast a projectile from a cannon. The time of flight and range can be measured through the simulation, but for our purposes it is better to do these measurements without the computer. You will need find a stopwatch (phone, google, wristwatch) and to utilize the on-screen measuring tape.
1. Experimental Setup

1.1. Open the simulation in your browser of choice and select Lab.

1.2. Adjust the angle of the cannon as instructed for each launch by dragging the yellow muzzle.

1.3. For practice, launch a few projectiles and try to land in the center of the landing pad. Feel free to adjust any parameter in the settings; there will be a reset before the lab activities.

NOTE: You may have to the ability to adjust the height of the cannon by dragging the platform. For the other activities it is important that the launch platform should be at the same height as the landing pad for one-level projectile motion (i.e. $y = y_0$).

2. Determining $v_0$

In this section, your task is to take some manual measurements of the projectile’s time-of-flight and range in order to verify through calculation the initial velocity of the projectile using the two models on p. 2 (Eq. 5 and Eq. 6, respectively). With this value for $v_0$, you will later go on to test the predictive power of these models. Although we are setting $v_0$ in the simulation, you will find that when measuring manually that your results may vary.

2.1. Always begin by resetting the simulation.

2.2. Change the projectile being launched from a cannonball to a baseball. Set its Diameter to 0.40m...it’s a big baseball.

2.3. Set the initial velocity to 30 m/s and the launch angle to 57 degrees. Record these in your notebook.
2.4. Launch the projectile by clicking the red cannon button.

2.5. Measure the time of flight and range between when the ball leaves the cannon and when it strikes the ground. Do not use the simulation program tools to measure the time, as mentioned in the introduction, use your phone, watch, or google stopwatch. Measure distance with the on-screen tape measure not the other measurement tools. Record these values in your notebook, as well as the nominal initial velocity, and angle for each attempt.

Tip: Click the magnifying glass (with the - sign as shown on the right) in order to view all of the simulation.

2.6. Repeat steps 2.1-2.5 two more times to decrease random uncertainty.

2.7. After the three trials have been completed. Estimate the uncertainty attributable to your time-of-flight and your range measurements. Make sure to explain your logic. Which measurement (time-of-flight or range) is more precise?

2.8. Although we can clearly see the nominal initial velocity, let’s see how it compares to manually measured data. Use Eq. 5 and Eq. 6 to calculate two separate values for \( v_0 \) according to the time-of-flight and range measurements, respectively. Call the value using the time-of-flight data \( v_0^{(t)} \) and the value using the range data \( v_0^{(r)} \). Propagate the uncertainty for both calculated values of \( v_0 \) from your measured values that you estimated in 2.5. Which of these \( v_0 \) values for the initial projectile velocity is more precise? Which is more accurate? Why?

3. Predicting Future Behavior

Based on your values for the initial velocity parameter that appears in the models described by Eq. 5 and Eq. 6, the models can now be used to predict future observations. (You practiced this in your pre-lab assignment.)
3.1. Write out Eq. 5 and Eq. 6 in your notebook with the values from step 2.6 for $v_0^{(t)}$ and $v_0^{(r)}$ plugged in to their respective models.

3.2. Draw in your notebook two separate graphs of each equation with your $x$-axis spanning from $0^\circ$ to $90^\circ$. The graphs need to be large enough for you to accurately extrapolate values from them. If you prefer to work in units of radians, your axis should span from $0$ to $\pi/2$ rad. Make sure to mark the data point you have already acquired at a launching angle of $57^\circ$. Why do you think we choose to limit the launching angles to this range?

3.3. Prepare two tables each with two columns adjacent to each of your drawn graphs that look like those shown below. From your drawn graphs, infer values for time-of-flight and range at each of the angles shown.

<table>
<thead>
<tr>
<th>Launching Angle (deg.)</th>
<th>Time-of-Flight (s)</th>
<th>Launching Angle (deg.)</th>
<th>Range (m)</th>
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</thead>
<tbody>
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3.4. At what launching angle should the time of flight be maximized? At what launching angle should the projectile range be maximized?

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4. Comparing Prediction to Observation

Now that you have used the models (in Eq. 5 and Eq. 6) to predict time-of-flight and projectile range values for a series of launching angles, you will produce measurements using those launching angles to test the predictions.

4.1. Repeat steps 2.1-2.5 to collect time-of-flight and range data for all the launching angles listed in the tables in your notebook. Create new tables in your notebook where you can record the measured data. Make sure to
average each set of three measurements for each launching angle and to estimate uncertainties for each recorded measurement.

- Explain why you do not need to collect data for 0° and 90°.

4.2. Compare the tables filled out in step 4.1 with those in step 3.3. How accurately were you able to predict the time-of-flight and range for each of the prescribed angles? Be sure to include the uncertainty values in your comparisons (reference the Uncertainty Analysis Guide if needed).

4.3. Create a table each for “Time-of-Flight vs. Launching Angle” and “Range vs. Launching Angle” in Excel. Input your measured values into the respective columns.

4.4. In your notebook, write down the mathematical models that you would use to represent Eq. 5 and Eq. 6 (you did this in your pre-lab). Make sure to explain what terms in the physical model the parameters in your mathematical model refer to.

4.5. We can put our data into a form that Excel can properly fit by “linearizing” our data. To do this, create a new column in your Time-of-Flight data set for sin(θ) and one in your Range table for sin(2θ). Note that Excel expects angles to be in radians, so it may be easier to create a column where you convert degrees into radians before calculating sin(2θ). The picture to the right shows a sample of what your time of flight table might look like (Note that the data in this picture is intentionally inaccurate, so your TA will give you a bad grade if you simply copy it).
4.6. Now, from your tables for “Range vs. Angle” and “Time of Flight vs. Angle” have Excel generate scatter plots with Range or Time of flight on the y-axis and $\sin(2\theta)$ or $\sin(\theta)$ on the x-axis. Add a linear trend line for each data set and be sure to have Excel “show equation on chart.” It may also be beneficial to check the “Set Intercept” option here too.

4.7. The initial hypotheses provided to you in this lab are stated mathematically in Eq. 5 and Eq. 6. Compare your fit parameters from 4.6 to these two equations. How accurately do your fit parameters match your launch velocity?

4.8. Screenshot both graphs containing your measured data and your mathematical fits. Print or draw these for your report and show them to your TA.

4.9. In the analysis you performed today, you measured initial conditions and final conditions and then inferred information about projectile motion. What assumptions about the object’s trajectory allow you to ignore the object’s behavior in-flight?

Finally, write a short summary of today’s activities. In it, please discuss:

- Why is it valuable to have practice observing, measuring, and predicting an object in projectile motion?
- The two “separate” models used in today’s experiments are both based on the same related set of kinematic equations. Why did the models not predict your measurements with the same degree of accuracy?
Projectile Motion

- In the projectile motion tests from today, was the acceleration vector ($\mathbf{a}$) ever perpendicular to the velocity vector ($\mathbf{v}$)? Parallel to $\mathbf{v}$? Explain.
- What information do you gain by conducting graphical analysis to compare measurements to prediction? Why can’t you just calculate the numbers from each measurement and compare them to the predictions based on the initial conditions of the flight?
Pre-Lab Assignment (1 point)

1. Read through the “Measurements and Experimental Results: Graphical Analysis” document found on your lab section’s iLearn page. Make sure to follow the given example throughout the different sections in the document – this will help you understand the general protocol for conducting a proper graphical analysis, which you will practice in the questions below.

2. Your initial hypotheses to describe projectile motion are given to you in Eq. 5 and Eq. 6. Write down the mathematical models that you would use to represent Eq. 5 and Eq. 6 (reference p. 2 of the Graphical Analysis guide). Make sure to explain what terms in the physical model the parameters in your mathematical model refer to.

3. Graph these two functions by hand on separate graphs with launching angle on the horizontal axes and time-of-flight and range on the vertical axes, respectively. Use $v_0 = 3.92$ m/s.

4. Copy the tables below and fill in the approximate values for time-of-flight and range by reading them off of your hand-drawn graphs. (It would be just as easy to calculate them individually, but it is an important exercise to infer information from graphed data, so please do so!)

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