Objectives

This week, you will again examine wave mechanics as you did last week, but this time from the perspective of the oscillating medium. You will develop a visual and mathematical understanding of resonant phenomena, and you will then proceed with comparing your theoretical understanding with physical observations.

For the following activities, you will use a physics simulation program. Visit: [https://uglabs.physics.ucr.edu/](https://uglabs.physics.ucr.edu/) for lab downloads and links.

1. What is Resonance?

Recall from last week our mathematical model of the propagation of waves:

\[ y(x, t) = y_0 \sin \left( \left( \frac{2\pi}{\lambda} \right) x - (2\pi f) t + \phi_0 \right) \]

The equation in this form predicts the position \( y \) of a single point in the oscillation direction of a propagating harmonic wave for a given position \( x \) along the propagation direction of the wave and a given time \( t \). This wave equation is written in terms of the amplitude \( y_0 \), wavelength \( \lambda \), frequency \( f \), and phase \( \phi_0 \). All these parameters describe characteristics of the wave’s motion, but what happens when we change the medium in which the wave is propagating?

Recall from last week our observation of standing waves on a string: the velocity of wave propagation is dependent on the static properties of the oscillating object. Specifically, in the case of a string being suspended horizontally with an applied tension:

\[ v = \sqrt{\frac{T}{\mu}} \quad \text{(Eq. 1)} \]
Resonance

$T$ is the tension in the string (generated by a mass hanging on one end of the string) and $\mu$ is the mass-per-unit-length of the string.

For the following questions, imagine the apparatus shown below. It is similar to the apparatus you used in the simulation last week, however, instead of a clamp applying tension to one end of the string, we have a mass hanging over a pulley.

1.1: Suppose the apparatus above had a string of $L = 68 \text{ cm}$ and you found the $n = 2$ harmonic at $20 \text{ Hz}$. Calculate the wave velocity, referring to last week’s lab if necessary.

1.2: Suppose the hanging mass on the string was $\sim 30.2 \text{ g}$ and the mass-per-unit-length of the string was $\sim 1.60 \times 10^{-3} \text{ kg/m}$. Calculate what the wave velocity would be for these values using Eq. 1. The two velocity values aren’t quite the same (because no measurement is ever perfect), but they should be the same within measurement uncertainty!

→ That these values are the same tells us a very important bit of information: that we can describe wave motion either by understanding the propagation of the wave or by understanding the intrinsic properties of the oscillating medium.

This is only valid when the wave is in resonance with the oscillating medium. Recall in standing waves on a string, the oscillation was irregular at non-harmonic frequencies. At the harmonic frequencies, the wave was in resonance with the string, and at other frequencies, the wave was out of resonance with the string.
Resonance

You can think of every object as having some natural frequency at which it wants to vibrate. This “vibration” resonance is only one form, you can have a resonant system with any kind of oscillation, like optical resonance (lasers), gravitational resonance (planets in orbit), electrical resonance (radio antennae), and so on.

1.3: Write down a definition of “resonance” as it makes sense to you.

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2. Mathematical Description of Resonance

Let’s change the physical system we’re thinking about again (I repeat – you can use the same wave mechanics to describe any oscillating system!). Let’s think about simple harmonic oscillation in a spring.

In the Week 1 lab, you studied simple harmonic oscillation in a “mass on a spring” system. Recall the relation between oscillation frequency and the static properties of the spring system:

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]

Compare this relation to Eq. 1 above. If we multiply both sides by some displacement (let’s call it \( x \)) and do some rearranging, we get:

\[ f \cdot x = \frac{1}{2\pi} \sqrt{\frac{k \cdot x^2}{m}} = \frac{1}{2\pi} \sqrt{\frac{k \cdot x}{m/x}} \]

This looks the same as the equation for describing standing waves in a string (Eq. 1). A speed \((f \cdot x)\) is equal to the root of a force \((k \cdot x)\) over a mass-per-unit-length \((m/x)\). In the string, the material properties determining the resonant frequency were the string tension and the mass-per-unit-length. In the spring system, the material properties are the spring constant and the hanging mass.

You have already verified this relation experimentally in the Simple Harmonic Oscillation lab. This is extremely powerful, though, because it tells you that anything that acts like a spring – i.e. undergoes simple harmonic oscillation – has a resonant frequency defined to be something like this mathematical form! And since we can describe each of these systems using simple harmonic oscillation, there must be some condition at which each system is in resonance.
2.1: Describe in your notebook how the following systems can be modeled like a spring undergoing simple harmonic oscillation:

- A diving board
- A planet orbiting a star
- A guitar string
- The human circulatory system

2.2: Describe what might happen in one or two of the systems above if they were operating out of resonance. Turns out it’s a pretty important phenomenon!

*Thought Experiment:* In the “Introduction to Waves” lab, you studied standing waves in a one-dimensional system. This standing wave pattern consisted of two superimposed waveforms – an excited and reflected wave – that at specific wavelengths ($\lambda = 2L/n$), would constructively and destructively interfere in a stable equilibrium pattern. But there is no reason this kind of stable equilibrium can only exist in this simple one-dimensional system! Imagine a two-dimensional system in which a sinusoidal waveform is excited at the exact center of a circular plate. The standing wave pattern is established when the excited wave two-dimensional waveforms for a stable equilibrium pattern of constructive and destructive interference with the reflected waves from the edge of the circular plate (a good way to visualize this might be to consider the behavior of ripples in a pond reflecting from a barrier that forms an exact circle around the point where the ripples were excited). Draw what you expect the standing wave patterns to look like in this two-dimensional system for increasing resonant harmonics, $n$. Explain what characteristics of the oscillating plate affect the value of those resonance frequencies, and what geometric properties of the plate affect the patterns.

*NOTE:* These two-dimensional oscillating plates are called “Chladni plates” – I recommend looking up some YouTube videos or descriptions of a Chladni plate in operation. It’s really cool!
We thought a little bit last week about sound waves and their propagation through air. By definition, a sound wave is an oscillation between compression and rarefaction longitudinally through a medium (like air) to generate alternating regions of high pressure and low pressure.

We know that pressure ($P$) is a force distributed over some area, so maybe we can approximate this pressure as a sort of “elasticity” term (i.e. spring constant) for air. We then need a mass term that accounts for air mass distributed in three dimensions – that’s the density ($\rho$). Now, assuming our ‘spring model’ of resonance developed above is valid, we might logically write:

$$v_{\text{sound}} = c = \sqrt{\frac{p}{\rho}}$$

Where we define ‘$c$’ to be the speed of sound in air. Even if this logically makes sense, we need to make sure it accurately models the physics of sound before we can claim it to be “correct”.

3.1: The first thing we should do is to check the units. Do the dimensional analysis to make sure the units work out.

3.2: The air pressure (called the ‘Barometric pressure’) and air density are measurable, but that is beyond the scope of this lab. These values change with local atmospheric conditions, but generally: $P_B = 101 \times 10^3 \text{ Pa}$ and $\rho_{\text{air}} = 1.23 \text{ kg/m}^3$. Calculate $c$ using the relation we reasoned out above. This is your theoretically determined value for $c$ (call it $c_{\text{th}}$).

Now we need to experimentally check if this value is right. We spent last week developing a model for the description of wave propagation, so let’s exercise that math and try to confirm our relation by measuring the speed of sound.

3.3: If you haven’t already, refer to the statement in the “Objectives” section of this lab to install the simulation.
3.4: The simulation contains the following:

a. A glass cylinder, with 10cm increments, filled with water. We will refer to this as the resonance tube
b. A speaker connected to a frequency generator (a piano).
c. Controls for the atmosphere gas.
d. Controls for the temperature of the water.
e. Controls for the water level.

Before beginning the lab, become familiar with the controls. Try adjusting the water level, generating different frequencies, adjusting the water temperature, and changing the atmosphere gas.

You may notice two additional functions: a slider to the right of the resonance tube and a box with a slash.

The slider will adjust the wave representation by increasing or decreasing frequency. Note that only the visual representation of the wave is affected, not the generated frequency.

Try clicking the box; a black bar will appear. You can use this bar as a marker throughout the lab by clicking and dragging the bar to the desired location on the resonance tube.

3.5: This system can be modeled like an open-closed resonator. Which end can we take to be “open” and which one to be “closed”? Why?
Resonance

3.6: The sound resonated from the tube is extended beyond the tube. We will need to include a correction of 15cm. This correction is considered a part of the standing wave even though it is outside the tube. Be sure to include it in any calculations involving the length of the resonating air.

3.7: From your pre-lab, you know how to relate the wavelength of a standing wave to the length of the resonator, and you also know \( c = \lambda f \). Using your value of \( c_{th} \) (step 3.2) and a frequency value of 261.63Hz, calculate the wavelength of the \( n = 1 \) mode for this system. Then calculate the resonator length, \( L \), at which \( n = 1 \) resonance will occur. Remember to include the correction!

What should physically happen in the system that will allow you to characterize when it is in a harmonic resonance vs. out of resonance? Explain.

3.8: Generate a frequency of 261.63Hz by pressing the C key on the piano.

*NOTE: Once you press the C key on the piano, the speaker, including your computer speakers, will play the note. Please either adjust your computer volume or the in-simulation volume control as necessary – please be careful!*

Slowly adjust the water level to the calculated length from step 3.6, then slowly adjust until you find a resonance condition. Record this value of \( L \). You may wish to mark this value using the bar markers. Comment on any differences between your predicted and measured values of \( L \).

3.8: Using your measured value of \( L \) and the frequency, calculate the speed of sound. This is your *experimentally determined* value – call it \( c_{ex} \). The value commonly used for the speed of sound in air at 20 °C is 343 m/s. Compare this with your experimentally determined value for \( c \).

3.9: Compare the experimentally determined value from 3.8 to the theoretically determined value you calculated earlier using our resonance model from
3.2. Discuss in detail the comparison between your $c_{th}$ and your $c_{ex}$. Is our resonance model valid? (Hint: It isn’t!)

3.10: We will now confirm that the open-closed resonator only allows for odd harmonics. With the amplitude knob turned to minimum, turn on the function generator and set the frequency to 329.63 Hz (E4 key). Calculate what the resonator length, $L$, should be for $n = 1, 2, 3, 4, 5$. Repeat the procedure from 3.8 to try to identify each of those harmonics. Discuss your observations.

Thought Experiment: We have studied the open-closed resonator in air. Let’s turn our focus to other gases: helium (He) and sulfur hexafluoride (SF₆). Helium is lighter than air. SF6 on the other hand is heavy, almost 5 time heavier than air. How would each gases speed of sound compare to that of air? Higher? Lower? The same? How do you expect the resonator to react in each gas? You may use the simulation to change the gas and finding the resonances for C4: 261.63Hz. Discuss your observations.

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4. Analysis

Let’s explore why the spring model in part 3 was invalid, starting mathematically. Comparing the commonly used value for the speed of sound in air ($c = 343 \text{ m/s}$) and the theoretically determined value of $c$ that you calculated using the not-quite-valid resonance model (you should have $c_{th} \approx 286 \text{ m/s}$), we can write:

$$c = \sqrt{\gamma} \cdot c_{th} = \sqrt{\gamma} \cdot \sqrt{\frac{P}{\rho}} = \frac{\sqrt{\gamma \cdot P}}{\rho}$$

4.1: We can define $\gamma$ to be some arbitrary constant value that satisfies the above equation. Solve for $\gamma$.

From a purely mathematical standpoint, then, it looks like we need to include some multiplicative factor ($\gamma$) inside our resonance calculation to make it valid. But what does this $\gamma$ term mean physically? Recall that within any closed system, the amount of energy is a sum of the system’s internal energy, work done on the system by its surroundings, and work done by the system on its surroundings.
Resonance

4.2: One form of energy that exists in every system but is difficult to quantify is heat. Think about how we formulated our spring resonance model. Did we account for the heat energy in the medium? Why do we need to?

4.3: Explain the mechanism by which heat energy can affect the phenomenon of resonance in sound waves propagating in air. (Hint: Think about what air is made of – atoms and molecules. Think about where energy can go in air that is different from the kinetic energy that atoms and molecules possess.)

The mathematical derivation of $\gamma$ is complex (we’ll skip that for now), but the result is powerful. We have a model to find the speed of sound in any medium:

$$c = \sqrt{\frac{\gamma P}{\rho}}$$

We can say “$\gamma P$” is something like the “spring constant” for a sound-oscillating medium (like air). And, more importantly, we have another valid application of our spring model of resonant wave oscillation.

Let’s conclude this lab by thinking about our spring model of resonance in another important context: in molecules and crystals. In the figure below, you see some different molecular configurations. These molecules increase in complexity from one dimension to three in the orientation of their molecular bonds.

Suppose the molecular bonds are springs. It is a really useful supposition because it allows us to visualize how energy is transferred through molecular bonds. And if molecular bonds are all springs, then there must be some fundamental resonant frequency and its incremental harmonics in the molecule!

4.4: Draw schematics of the configurations shown but replace the molecular bonds with springs: each bond has a spring constant ($k_n$) and each molecule has a mass ($m_n$). Describe how you might develop a model for calculating the resonant frequency for each of the three molecular configurations (you don’t have to work out the math!). This model describes “acoustic phonons”.

PHYS 2LC: Lab 3
Recall from the “Introduction to Waves” lab that it was easy to calculate the harmonic number \((n)\) and wavelength \((\lambda)\) of standing waves on a string by counting the number of antinodes \((\lambda_n = 2L/n)\). That was a system with nodes fixed at the end points. Today you will be working with a system that has one open end and one closed end \((i.e.\ a\ node\ fixed\ at\ one\ end\ and\ an\ antinode\ fixed\ at\ the\ other\ end)\). In this system: \(\lambda_n = 4L/n\).

Discuss why it is only possible to produce the odd harmonics in a system with one open end and one closed end (pictures will help!). Write down the relation between \(\lambda_n\) and \(L\) for the \(n = 1, 3, 5,\) and \(7\) harmonics, and calculate the wavelength and frequency of the fundamental resonant mode for \(L = 0.875\ m\) and \(f_7 = 686\ Hz\). Then calculate the wave velocity of this oscillation.

\[
\begin{array}{ccc}
\text{Harmonic Number:} & \text{Number of visible nodes:} & \text{Harmonic Frequency:} \\
1 & 1 & f = f_0 \\
3 & 2 & f = 3f_0 \\
5 & 3 & f = 5f_0 \\
7 & 4 & f = 7f_0 \\
\end{array}
\]