PHYS 40B: Lab 3
Simple Harmonic Oscillation
(Includes Pre-Lab Assignment)

Objectives

These lab activities will focus on the concepts of Hooke’s Law of elasticity and simple harmonic motion. Masses on a spring will be used to verify Hooke's Law and to measure oscillation periods in a simple harmonic oscillator. Additionally, a simple pendulum will explore another system which exhibits simple harmonic oscillation.

Visit [https://uglabs.physics.ucr.edu](https://uglabs.physics.ucr.edu) for this week’s simulations.

Introduction

Any force that always acts to bring an object back to an equilibrium position will produce harmonic oscillation. Such a force is known as a restoring force. In this lab you will explore two different systems that exhibit harmonic oscillation.

The most ideal kind of restoring force is that of a mass hanging on an ideal spring (or other elastic material). For any elastic material, such as a spring, that is stretched or compressed (displaced) from equilibrium by an amount, $x$, the restoring force is proportional to the amount stretched or compressed (displacement). This is a description of Hooke’s Law, or:

$$F = -kx \quad (1)$$

This relation implies that if we displace something away from its equilibrium position, it will bounce back in linear proportion to the distance it was displaced. The constant, $k$, is dependent on the system that was perturbed. For a very stiff system, $k$ will be large, and for a slack system, $k$ will be small.
Simple Harmonic Oscillation

Any system whose restoring force obeys Hooke’s Law (eq. 1), will exhibit what we call Simple Harmonic Oscillation. In today’s lab you will explore two systems that undergo simple harmonic oscillation. The first is the quintessential simple harmonic oscillator; a mass hanging on a spring. The second is a pendulum, which always experiences harmonic oscillation, but can be shown to experience simple harmonic oscillation when its angle of displacement is kept small.

For a mass hanging on a spring, neglecting the mass of the spring, the total force experienced by the mass is (see the Free Body diagram to the right):

\[ F_{\text{total}} = mg - kx = 0 \]

The displacement, \( x \), is measured from the equilibrium position of the spring. In the lab, you will vary the mass \( m \) and measure the corresponding displacement \( x \) for each mass. Since the hanging weight is equal to the spring force (i.e. \( mg = kx \)), a plot of \( mg \) versus \( x \) will have a slope of \( k \), the spring constant.

The Physics of the Simple Harmonic Oscillator:

In an elastic system (like the “spring-mass” system, for example), think about what happens when you extend a spring and then release it – it oscillates back and forth around its initial position until the oscillation eventually dies down and the system returns to its initial position. Let’s look at the physics describing this motion, starting from statements of Hooke’s Law:

\[ F = -kx \]

and Newton’s 2nd Law:

\[ F = ma = m \frac{d^2x}{dt^2} \]

We can combine them to find:

\[ \frac{d^2x}{dt^2} = -\frac{k}{m} x \]
Simple Harmonic Oscillation

This is a second order ordinary differential equation that describes the motion of a mass \((m)\) attached to a frictionless spring with spring constant \(k\). A general solution to this equation takes the form:

\[ x(t) = x_0 \cos(\omega t) \]

where \(x_0\) is the initial position and the angular frequency, \(\omega\), must be:

\[ \omega = 2\pi f = \frac{k}{\sqrt{m}} \]

The cosine function that appears in the general solution has an extremely important physical consequence: *the motion of the mass on the spring will oscillate after being perturbed by some displacement \(x\).* The cosine function undergoes a complete cycle of oscillation in the amount of time it takes for the argument \((\omega t)\) increases by \(2\pi\) radians (or when \(ft = 1, 2, 3, \ldots\)). So, starting at \(t = 0\), the first oscillation will be completed in a time \(T\) such that \(\omega T = 2\pi\), or:

\[ T = \frac{2\pi}{\sqrt{\frac{k}{m}}} \tag{2} \]

where \(T\) is called the period of oscillation. If the spring is stiff (large \(k\)), the period would be small. Similarly, a larger mass leads to a longer period of oscillation. In this lab, you will measure oscillation period as a function of hanging mass. So a plot of \(T\) versus \(m\) will be a square root curve going through the origin with a fit parameter of \(A = \frac{2\pi}{\sqrt{k}}\). You will test this prediction to see if it accurately describes the “mass on a spring” system.

**The Physics of a Simple Pendulum and the Small Angle Approximation:**

A simple pendulum consists of two main components: (1) a “massless” string suspended from a pivot point, and (2) a mass or “bob” hanging from the end of the string. A pendulum, like a mass on a spring, can exhibit harmonic oscillation. It can be shown (see 3.1) that the restoring force for a simple pendulum is

\[ F = -mg \sin \theta \tag{3} \]
Simple Harmonic Oscillation

where $m$ is the mass of the pendulum bob, $g$ is the acceleration due to gravity, and $\theta$ is the angle the pendulum string is from vertical.

For small angles of displacement, one can approximate the pendulum as if the bob was moving horizontally back and forth (i.e. one can ignore the vertical component of the motion). Recall the definition of the sine function in relation to a right triangle:

$$\sin \theta = \frac{x}{L}$$

where $x$ is the side of a right triangle opposite the angle and $L$ is the length of the hypotenuse of the same right triangle, or, in the case of a pendulum, $L$ is the length of the pendulum and $x$ is the horizontal distance from the pendulum bob to the vertical line running through the pendulum’s pivot point. This means that we can rewrite equation 3 as

$$F \approx -mg \frac{x}{L}$$

for small angles of displacement. This is known as the small angle approximation for a pendulum. While a pendulum normally only experiences harmonic oscillation, at small angles, a simple pendulum quite accurately approximates simple harmonic oscillation.
1. Varying Masses on a Spring to Confirm Hooke’s Law

The simulation for Hooke’s Law contains four activities (Intro, Vectors, Energy, and Lab). For the purposes of our experiment, open the activity entitled “Lab.” An image of the Colorado PhET for this week’s lab is shown below:

1.1: Notice the weights and mass control in the simulation. The orange mass labeled “100 g” will be used to generate a force on the spring.

*It is important that you do not change any settings unless instructed.

1.2: Check the box next to the natural length feature in the sidebar. A dashed blue line will appear. Drag the ruler over to the spring and align the 0 cm end at the same height as the dashed blue line.
1.3: Hang the mass from the bottom of the spring. Once the spring oscillations settle, record the height of the loop at the bottom of the spring as measured on the ruler. This is the “equilibrium position.”

There are several tools in the sidebar (shown below) to assist you in making accurate measurements.

For Part 1, do not change the settings for gravity or damping. To revert back to the original settings, press the reset button (圆形图标)。

1.4: Make measurements of the spring elongation as a function of weight.
1.4.1: Enter your data into Excel in the following format:

<table>
<thead>
<tr>
<th>Added Mass (kg)</th>
<th>Added Force = mg (N)</th>
<th>Displacement = x (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.050</td>
<td>0.49</td>
<td></td>
</tr>
<tr>
<td>0.100</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td></td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td></td>
</tr>
</tbody>
</table>

1.4.2: Increase the mass in 50-gram increments up to 250 grams.

1.4.3: Measure the displacement of the loop for each mass and record its value.

1.5: You will now analyze your data graphically to compare the Hooke’s Law model with your data.

1.5.1: Enter your force data into the “Force” column to complete the table.

1.5.2: Now look at Eq. 1. Which data corresponds to the equation? Which mathematical model will you use to fit your data graphically? Explain what fit parameters you will use and what physical terms in Eq. 1 they relate to.

1.5.3: Plot the force versus displacement data in Excel and select the fit function you determined in 1.5.2. Refer to the figure below for an example of adding a trendline to a plot. To view the fit function for your data, click on “More Options...” in the Trendline menu and check the box for “Display Equation on chart.” Use the fit function to determine the fit parameter and the spring constant, $k$, for your spring. Add the plot to your report.
Q1: Does Hooke’s Law adequately describe your measurements? How accurately was $k$ measured? Qualitatively discuss any random and systematic uncertainties that affected your measurements. How could you make your measurements more accurate and/or precise? How could you adjust the Hooke’s Law model to make it more accurately model your data?

Q2: Most scales use a spring, or spring like mechanism to measure weight. Now that you’ve explored Hooke’s Law, how do you think you would design a simple scale using a spring whose spring constant, $k$, you know?

2. Simple Harmonic Oscillation

2.1: In the following activities, you will observe a mass-spring system under simple harmonic oscillation. You will then use the data you collect to reinforce the ideas from the pre-lab assignment. Begin by changing the damping slider to “None.”

2.2: Set the 100g mass onto the spring. Wait for the spring to settle again. You can speed up by pressing the stop sign ( ).
NOTE: For this activity, slowing down the simulation and making use of the pause button will be helpful while you’re taking data.

2.3: You will now be using the on-screen timer and the ruler to record displacement as a function of time. Drag both items into the experiment area. Line up your ruler such that 0 cm is in line with the loop at the bottom of the spring. This will be your reference for “zero” each time you reset the spring-mass system.

2.4: To begin recording data, pause the simulation then click on the pause button on the timer. Place the mass on the spring and drag it up so that the bottom of the spring lines up with 0 cm. When you press the “Play” button on the simulation, the system will bob up-and-down while the timer will begin recording. Pause the simulation to freeze time then measure and record displacement and time. Restart for a fraction of a second, pause again, then record the new displacement and time. Repeat this so that several measurements are recorded for each period of oscillation. Data collection can stop once at least three full periods have been recorded.

2.5: Plot the data in Excel using a Scatter Plot with Smooth Line. Record the Period by taking the difference in position between two troughs or crests (as illustrated above). The angular frequency of the spring-mass system is related to the oscillation period by $T = \frac{2\pi}{\omega}$. Determine $\omega$.

2.6: Repeat the period measurement for added mass in 50 gram increments up to a total of 300 grams. For each iteration of this process, take the mass off the spring and adjust the mass on the platform. Then, put the new mass
onto the spring and reset the oscillation. Determine the period and its uncertainty each time. Record your data in your report.

3. The Simple Pendulum

3.1: By drawing a free body diagram for a simple pendulum and breaking the forces into components, show that the restoring force on a pendulum obeys equation 3 if we ignore friction and air resistance. (*Hint:* the net restoring force for a pendulum is the component of the sum of the forces that is perpendicular to the pendulum’s string.)

3.2: Reconsider the section in the introduction concerning the small angle approximation. Based on equation 4, what is the effective “spring constant,” $k$, of a pendulum for small angles of displacement? (*Hint:* compare equations 4 and 1.)

3.3: For small angles, find an expression for the period of a pendulum. (*Hint:* compare the value of the “spring constant” you found in 3.3 and equation 2.)

3.4: Open the pendulum simulation. And move to the “Intro” tab. Set the length of the pendulum to 0.25 m and the gravity to “Earth.” Make sure the friction is set to none.
3.5: Start the pendulum moving by displacing it by no more than 5 degrees, then use the stopwatch provided by the simulation (or one of your own) to measure the period of the pendulum. Do so by measuring the time it takes for the pendulum to return to its maximum ten times then divide that value by ten to get an experimental value for the period. Be sure to estimate your uncertainty.

3.6: Compare your measured period to the period you calculate based on the equation you derived in 3.3.

3.7: Repeat steps 3.5-3.6 for pendulum lengths of 0.5 m and 1 m.

Q3: How might you use a pendulum to measure the acceleration due to gravity on an unknown planet?

Q4: Under what conditions is Hooke’s Law invalid? Does a spring always exhibit simple harmonic motion? Does a pendulum? What is simple harmonic oscillation?
Pre-Lab Assignment (1 point)

1. Draw by hand a graph of displacement vs. time for an oscillating spring with spring constant, \( k = 0.64 \) N/m, and hanging mass, \( m = 0.25 \) kg.

2. From your graph in #1, draw graphs for the velocity of the moving end of the spring, \( v \), as a function of time and also the acceleration of the moving end of the spring, \( a \), as a function of time (these don’t have to be drawn numerically precisely, but keep your \( t \)-axis scale the same for all three graphs).

3. Discuss the implications of the relation between position, velocity, and acceleration in a simple harmonic oscillation system.