PHYS 40C: Lab 6
Time-Dependent Electrical Circuits
(Includes Pre-Lab Assignment)

Objectives

These activities will focus on circuits with voltages and currents that change with time \( (V(t)) \). You will work with RC circuits and examine the charging and discharging behaviors of a capacitor and use that information to determine the time constant and capacitance for the specific circuit.

For the following activities, you will use two physics simulation programs. Visit: https://uglabs.physics.ucr.edu/ for lab downloads and links.

Introduction

In previous labs, we have discussed circuits with properties that did not change with time. We assumed that the voltage drop across components was constant in a given configuration. This week, we will be studying time-dependent RC (resistor-capacitor) circuits.

The simplest example of a time dependent circuit is the charging or discharging of a capacitor in an RC circuit. The voltage across a charging capacitor with capacitance, \( C \), is given as a function of time, \( t \), by:
\[
V = V_0 \left( 1 - e^{-t/RC} \right) \quad (\text{Eq. 1})
\]
where \( R \) is the resistance value of a resistor and \( V_0 \) is the value of a voltage placed in series with the capacitor. Such an arrangement is called a Resistor-Capacitor (RC) circuit. The product \( RC \) has units of time and is referred to as the time constant, \( \tau \), of the circuit. \( \tau \) represents the amount of time it takes for the capacitor to charge to 63% of its final value. The voltage across a discharging capacitor is given by:
\[
V = V_0 e^{-t/RC} = V_0 e^{-t/\tau} \quad (\text{Eq. 2})
\]
Where \( \tau \) represents the amount of time it takes for the capacitor to discharge to 37% of its initial value. A good rule of thumb to use is that is takes about \( 5*\tau \) for a capacitor to look like it is either fully charged or discharged.
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Simulation 1

Resistance = 2.00 ohms
Capacitance = 1.00 F

Simulation 2
1. Charging and Discharging a Capacitor

1.1: Open Simulation 1. You should see the following page.

1.2: Before we use the simulation, refer back to Eq. 1 and Eq. 2. Graph by hand the shape of the Voltage vs. Time curve for both charging and discharging of a capacitor in an RC circuit. Explain why the curves look the way they do. (Numbers aren’t important here, just identify the correct curve.) To charge the capacitor in the circuit in this simulation, you will be using 6 V from the battery. What is the maximum voltage that the capacitor can be charged to?

1.3: Click “Reset.” The voltage of the charging capacitor should be shown on the Voltage vs. Time plot. Once you see that the capacitor is fully charged, click the “Battery removed” button to begin discharging the capacitor. When the capacitor is fully discharged, click “Pause.” Comment on how this plot compares to the one you drew in step 1.2.
1.4: Make a copy of the Voltage vs. Time plot in your notebook. Also include a drawing of the circuit diagram with all parts labelled (including their values).

1.5: Predict how changing the resistance will affect the circuit’s behavior. What about the capacitance? Try three different resistor-capacitor combinations. Record which values you tried, and for each combination, describe how the charging/discharging times were affected. Can you think of the resistor-capacitor value that results in the shortest charging/discharging time?

2. Constructing and Analyzing RC Circuits

2.1: Open Simulation 2 from your course materials webpage. Construct the circuit shown below and double-check it before continuing. Click the box next to the stopwatch to bring it on-screen. You can change the values of the resistor and capacitor by clicking on them and adjusting their sliders.

2.2: Close S1 and examine the screen on the voltmeter to get an idea of the charging time. Once the voltmeter shows that the capacitor is sufficiently charged, open S1 then close S2. Examine the discharging behavior on the voltmeter. This step should give you an idea of the timescale for this
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circuit’s charging/discharging cycle. Make sure the capacitor is completely discharged before proceeding to 2.3.

2.3: Pause the simulation by clicking the pause button at the bottom of the screen. Press “Start” on the stopwatch then open S2 and close S1.

2.4: Quickly pause the simulation, record values for time and voltage, then start the simulation again. Take 20 data points in this way as the capacitor charges. Once charged, pause the simulation, and reset the timer. Open S1 and close S2 then take 20 more datapoints for the discharging capacitor. Enter your data into an Excel spreadsheet then make a plot for Voltage vs. Time.

2.3: You may need a few attempts to get a clear plot of the charging behavior of the capacitor. To repeat an attempt, close S2 and discharge the capacitor fully then reset your stopwatch. Repeat steps 2.3 and 2.4. Does the plot match the curve you drew for capacitor charging behavior in part 1.2?

2.4: Once you are happy with your Voltage vs. Time data, create two separate plots for your charging and discharging data. To properly fit the charging curve, we must rearrange Eq. 1:

\[
\frac{V_0 - V}{V_0} = e^{-t/RC}
\]

or

\[
V' = e^{-t/RC}
\]

Use a fresh column in Excel to calculate \(\frac{V_0 - V}{V_0}\), which we will now refer to as \(V'\), for each voltage datapoint you collected. Plot \(V'\) vs. \(t\). Which fit should you apply to get the time constant, \(RC\)? Record the time constant from the graph of the charging capacitor. Include a copy of your plot and fit equation in your lab report.

2.5: Since the formula for a discharging capacitor is already in a form the Excel can automatically fit, apply the appropriate fit to your discharging plot and
record the time constant. Include a copy of your plot and fit equation in your lab report.

2.6: Is the time constant the same for both the charging and the discharging curves? Can you explain any discrepancies?

2.7: Confirm the capacitance value of your capacitor, \( C \), using the time constants found above and the value of \( R \) from the simulation. Estimate the uncertainty for this measurement based on systematic and random uncertainties that you have observed in your experiment.

- \textit{Thought Experiment:} How might we change the value of the time constant associated with charging/discharging capacitors by varying the circuit we used here? How might we change them by redesigning-the capacitor itself (recall the parallel plate capacitor design we studied in an earlier lab)? Explain.

3. Multiple RC Circuits in Series

Now that you are familiar with the behavior of simple RC circuits, let’s study a more complex system – what happens when we put two RC circuits together?

3.1: First, formulate a hypothesis. In part 2, you observed charging and discharging of a capacitor in an RC circuit that adhered to the model described by Eq. 1 and Eq. 2. How do you think that model will change if we add a second RC circuit? Will the time constant change? Will the charging and discharging behavior change entirely? Etc.

3.2: Now add another RC circuit to your circuit so that the two RC configurations are in series. In part 2 above, you used a voltmeter to measure the voltage drop across the capacitor as a function of time. Set up one voltmeter in your circuit so that you can measure the voltage drop across the first capacitor, and the second voltmeter so that you can measure the voltage drop across the second capacitor. The circuit diagram for this setup is shown below.
3.3: After acquiring 20 data points of charging and 20 data points of discharging behavior for both capacitors in the circuit above, use Excel to generate a plot of your data. Discuss the behavior you observed and whether or not it fits with your hypothesis. If you were to measure the voltage drop across both capacitors (like in the circuit shown below), describe and draw what you would expect that to look like. Is your hypothesis still the same as you described in 3.1?

3.4: Move the leads on the voltmeter to match the circuit schematic above. Obtain 20 data points for charging and discharging of the circuit, then plot
your data in Excel. Describe what you observe. Does it match your hypothesis from 3.1 or 3.3? Why or why not? Can you explain the behavior that you observe?

- **Thought Experiment:** Explain what is physically happening to the charge in the RC circuit during charging and discharging cycles (i.e. Why does the capacitor charge/discharge quickly at first but slows down with more time? Where is the charge going?). In the DC Electrical Circuits lab, we used the analogy of water flowing through a pipe to visualize Ohm’s law – can you extend that analogy to the RC circuit?
Pre-Lab Assignment (1 point)

1. A 7.5 F capacitor is charged to 9.0 V and placed in the RC circuit shown. At time $t = 0$ s, the switch is closed, allowing the capacitor to discharge into the 250 $\Omega$ resistor.

   a. What is the time constant (in seconds) for this RC circuit?

   b. After how many time constants is the charge on the capacitor one-fourth of its initial value?