PHYS 40C: Lab 8
Self-Inductance and AC Circuits
(Includes Pre-Lab Assignment)

Objectives

These lab activities will focus on the concepts of self-inductance in coils and impedance in AC circuits. You should read all the steps in each part before you start. Work in your assigned groups and maintain a collaborative and communicative team.

You will determine the impedance, $Z$, of two coils by measuring the \textit{rms} (root mean square) values of the voltage and current on an AC voltage and AC current meter (the \textit{rms} value is related to the peak voltage $E_m$ and current $I$ as $V_{rms} = \frac{E_m}{\sqrt{2}}$ and $I_{rms} = \frac{I}{\sqrt{2}}$). You will also assemble several circuits with inductors, resistors, and a capacitor, and determine the phase angle $\phi$.

For the following activities, you will use two physics simulation programs. Visit: https://uglabs.physics.ucr.edu/ for lab downloads and links.

Introduction

Self-Inductance

A simple coil of wire (a solenoid) can be described electrically by its self-inductance. That is, the magnetic flux a solenoid generates within itself when a current is applied. For an ideal solenoid of $N$ turns, cross-section area $A$ and length $l$, the inductance is given by

$$L = \frac{\mu_0 N^2 A}{l}$$
A real inductor will also have some intrinsic resistance and capacitance in the wire, so that it can be modeled as an ideal inductance $L$ in series with a resistance $R_L$ and in parallel with a capacitance $C_L$. 

In an AC circuit with an angular frequency $\omega = 2\pi \nu$, the impedance of the real inductor will thus be

$$Z = \frac{V_{rms}}{i_{rms}} = \sqrt{R_L^2 + X_L^2} = \sqrt{R_L^2 + \omega^2 L^2}$$

However, since we are working with computer-simulated inductors, our inductors will have no intrinsic resistance, leaving us with

$$Z = \frac{V_{rms}}{i_{rms}} = X_L = \omega L$$

$X_L = \omega L$ is the called the “reactance” of the inductor. Physically, the reactance is the opposition of the component to a change in current or voltage. In inductors, it’s a result of Faraday’s Law (“nature abhors a change in magnetic flux”). Direct measurement in the lab of $V_{rms}$ and $I_{rms}$ will allow calculation of $X_L$.

**Series RLC Circuit**

In a series RLC circuit driven by an AC source, the impedance ($Z$) of the circuit is given by:

$$Z = \frac{E_m}{I} = \frac{V_{rms}}{i_{rms}} = \sqrt{R^2 + (X_L - X_C)^2}$$

where $E_m$ is the amplitude of the applied emf, $V(t) = E_m \sin \omega t$, $I$ is the amplitude of the current, $i(t) = I \sin(\omega t - \phi)$, and where $X_L = \omega L$, $X_C = \frac{1}{\omega C}$.
and $R$ are the reactances and resistance of the element(s). The relative phase difference between the voltage and the current is given by the angle $\phi$, where:

$$\tan \phi = \frac{X_L - X_C}{R}$$

The relative phase difference is equivalent to a relative time delay $\Delta t$, through the equivalence

$$i(t) = I \sin(\omega t - \phi) = I \sin \left[ \omega t - \omega \left( \frac{\phi}{\omega} \right) \right] = I \sin(\omega t - \omega \Delta t).$$
1. Self Inductance

1.1: Open Simulation 1 and set up the circuit shown in the schematic below.

![Circuit Schematic 1]

1.2: Close the switch and describe your observations. Describe what effect the inductor has on the current. Open the switch and again describe your observations. Opening the switch effectively cuts off the battery from the rest of the circuit. Why do you think current was still able to flow after opening the switch?

1.3: Press the reset button (idebar) to clear the circuit then build the circuit shown below.

![Circuit Schematic 2]
1.4: Describe what happens to the light bulb when you close the switch. From your observations of the light bulb’s behavior, sketch what you think the voltage vs. time plot for the bulb would look like in your notebook. After you have made your prediction for the voltage vs. time plot, use the voltage chart tool (shown at right) to observe how the voltage across the light bulb changes with time. How does this compare to your prediction?

1.5: Open Simulation 2. This simulation has a much wider variety of circuit components and will give you access to a useful tool called the oscilloscope, or “scope” for short.

1.6: Create a blank circuit by clicking “Circuits” -> “Blank Circuit.” Create an AC voltage source by clicking “Draw” -> “Inputs and Sources” -> “Add A/C Voltage Source (2-terminal).” Set the max voltage to 6 V and the frequency to 60 Hz. Create a switch, 50 mH inductor and 100 Ω resistor then place them in series with the AC source as shown in the figure below.

1.7: We will now use some oscilloscopes to gather more information about this circuit. Scopes are powerful tools that allow the user to visually see time-dependent circuit behavior. Each scope display will show two curves, known as “traces,” which plot out voltage vs. time and current vs. time. A sample of what you should see is shown at right. You can use your cursor to hover over each trace and the scope will display information about the circuit in that particular point in time. Right-click the inductor and select “View on Scope” -- do the same for the resistor.
Self-Inductance and AC Circuits

NOTE: Sometimes the scope won’t display both voltage and current traces. If this happens, try right-clicking the scope and selecting “Remove scope.” After removing the scope, right click the component and again select “View in Scope.”

1.8: Calculate \( V_{rms} \) and \( i_{rms} \) using \( V_{max} \) and \( i_{max} \) measurements taken from the scope. Change the inductor to Coil 2 = 250 mH then again calculate \( V_{rms} \) and \( i_{rms} \). Calculate the impedance of each coil. Use your impedance calculation to confirm the given value of \( L \). Enter all data and calculations in a table of the form:

<table>
<thead>
<tr>
<th>Coil</th>
<th>( V_{rms} ) (V)</th>
<th>( i_{rms} ) (A)</th>
<th>( Z_L ) (Ω)</th>
<th>( L ) (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
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</tr>
</tbody>
</table>

- Thought Experiment: How are traffic lights triggered? You may have noticed that there are often circles or squares in roads where cars stop to wait at traffic lights. These are actually embedded wires with a small amount of current flowing through them. What happens when a metal loop (there are many in your car) comes to rest over the top of this current loop in the road? How does this trigger a traffic light to change?
2. AC Circuits

2.1: For this activity, we will continue using the circuit from part 1.8, shown below. Make sure you still have the scopes turned on for both the resistor and inductor.

2.2: Close the switch. Use the Source’s oscilloscope to measure the phase difference between the voltage on the resistor and the voltage on the inductor. You can do this by hovering your mouse over the scope and reading off the times. Assume that the 1 kΩ resistor is an ideal resistor, i.e., there is no capacitance or inductance ($Z_R = R$) and the voltage across the resistor should be exactly in phase with the current. If $i(t) = I \sin \omega t$ and $V_R(t) = IR \sin(\omega t + \phi)$, then $\tan \phi = \frac{X_L}{R}$.

The voltage across any other element(s) that have the same current will be given by $i(t) = I \sin \omega t$ and $V(t) = IZ \sin(\omega t + \phi)$, where the voltage leads the current by a phase angle $\phi$ given by $\tan \phi = \frac{X_L - X_C}{R}$. Compare your measured phase with the predicted phase by calculating the percent error.

2.3: Change the 100 mH coil to a 50 mH coil then repeat part 2.2. Comment on any discrepancy between measured and calculated phase.

2.4: Remove the 50 mH coil from the circuit then add a 5.0 µF capacitor in its place. Right click the capacitor again and select “View in Scope.” Repeat the
time delay and phase difference measurement of Part 2.2. Does the voltage across the capacitor lead or lag the current?

2.5: Repeat the time delay and phase difference measurement of Part 2.2 but replace the coil with a series connection of a 1.0 µF capacitor, a 50 mH inductor, and a 100 Ω resistor. Right click and select “View in Scope” as needed for each component. Compare the phase difference with what you expect by calculating the percent error.

- **Thought Experiment:** Consider the combination of Ohm’s law (AC) and the expression for impedance in an RLC circuit:

\[
i_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}}
\]

Noting that \(X_L = \omega L\) and \(X_C = 1/\omega C\), we know that for low frequencies, \(X_C\) is the dominant term, and for high frequencies, \(X_L\) is the dominant term. Analyzing this mathematically, though, there must be some “intermediate frequency” (call it \(\omega_0\)) for which the two reactances will be approximately equal and will cancel in the equation above, giving the simple relation \(i_{\text{rms}} = V_{\text{rms}}/R\). This will yield the maximum value for \(i_{\text{rms}}\) in the RLC circuit, operated at frequency \(\omega_0\), called the resonant frequency for the circuit. Derive a mathematical formula for \(\omega_0\) in an RLC circuit with inductance \(L\) and capacitance \(C\). Calculate \(\omega_0\) for the circuit used in Part 2.5. What does this resonant frequency represent physically? What happens when we operate an RLC circuit “off-resonance”?

Here are some useful hotkeys to take note of:

<table>
<thead>
<tr>
<th>Hotkey</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>w</td>
<td></td>
</tr>
<tr>
<td>shift+L</td>
<td></td>
</tr>
<tr>
<td>r</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td></td>
</tr>
<tr>
<td>s</td>
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<tr>
<td>Space</td>
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</tbody>
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Try out each hotkey to see what they do, then fill in the blanks on the table for each hotkey’s function.

2. Navigate to “Circuits” -> “A/C Circuits” -> “Inductors with Various Inductances.” Examine the three circuits that appear. The three plots at the bottom of the page show current vs. time for the three circuits. Describe the behavior of the circuits. Can you come up with a relationship between inductance and current?